ASSIGNMENT SET – I

Mathematics: Semester-I

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-105

Paper: Classical Mechanics and Non – Linear Dynamics

1. Show that the Coriolis force due to the rotation of earth deflects a vertically falling particle in northen hemisphere toward east and the deflection is proportional to $h^{\frac{3}{2}}$ for a given latitude where h is the height of the fall.
2. If the transformation equations between two sets of co-ordinates are

$P = 2(1+\sqrt{q}\cos p)\sqrt{q}\sin p, Q = \log(1+\sqrt{q}\cos p)$ 3.

then show that

(i) the transformation is canonical

(ii) and the generating function of this transformation can be put in the form $F = -(e^{Q}-1)^{2}\tan p$.

4. Show that the system $\ddot{x}+(2+3x^{2})\dot{x}+x=0$ is equivalent of the first order system
$\frac{dx}{dt} = y - x^{3}, \frac{dy}{dt} = -x + 2x^{3} - 2y$. Also show that the origin in the (x y) plane is asymptotically stable.
5. Consider the equilibrium configuration of the molecule such that two of its atoms of each of mass M are symmetrically placed on each side of the

third atom of mass m. All three atoms are collinear. Assume the motion along the modecules and there being no interaction between the atoms. Compute the kinetic energy and potential energy of the system and discuss the motion of the atoms.

6. Investigate the bifurcation of the dynamical system defined by
   $\frac{dx}{dt} = r + x^2$, where r and x are real.
7. Show that the path following by a particle in sliding from one point to another in absence of friction in the shortest time is cycloid.
8. State the fundamental postulates of special theory of relativity.
9. Prove that if the transformation does not depend explicity on time then the Hamiltonian represents the total energy.
10. Prove that the Work done by the force in displacing the particle from the position P_1 to P_2 is equal to the difference between the potential energies of the particle at those two positions
11. Suppose a rigid body is rotating about a fixed point. Prove that the kinetic energy is conserved throughout the motion.
12. Show that the rate of change of angular momentum is equal to the applied torque for a system of particles.
13. Write a brief note on phase portrait.
14. Define poisson brackets .show that it does not satisfy commutative property.
15. Prove that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz Transformation.

16. Construct the Routhian for the two –body problem , for which
   $$L = \frac{\mu}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$
17. A double pendulum consisting of two masses m_1 and m_2 oscillates in a vertical plane through small angles . m_1 is suspended from a fixed point by light inextensible string of length l_1 and $m_2$is suspended from m_1 by a similar string of length l_2 .
   State the number of degrees of freedom of the system and find the equations of motion by using Lagrange’s formulation.
18. Prove that the Poisson bracket of two constants of motion is itself a constant even when the constant s depend on time explicitly
19. Prove that the phase volume is invariant under canonical transformation..

20. Prove that the linear autonomous plane systems $\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$ is stable if $p^2 - 4q > 0$, q>0 , p<0.
21. Prove that Poisson braket is invariant under canonical transformation.
22. use Hamilton's equation to find the equation of motion of a simple pendulum.
23. State and prove the Jacobi Identity .
24. A body moves about a point O under no forces . The principal moments of inertia at O being 3A,5A and 6A. Initially ,the angular velocity has components $\omega_1 = n, \omega_2 = 0, \omega_3 = n$ about the corresponding principal axes .Show that at any time t ,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tanh(\frac{nt}{\sqrt{5}})$$

And that the body ultimately rotates about the mean axis.
25. Show that for conservative holonomic system, $\frac{\partial L}{\partial \dot{q}_j} = \int \frac{\partial L}{\partial q_j} dt$
26. Find the Euler's equation for the variational problem

Minimize $I[y(x)] = \int_0^1 (2x - xy - y')y' dx.$

27. Prove that the Poisson bracket of two constants of motion is itself a constant even when the constants depend on time explicitly.
28. If the generating function is known then show that the canonical transformation can be determined.
29. Discuss about stability of the following system of dynamical equations $\frac{dx}{dt} = -x + y, \frac{dy}{dt} = 4x - y$
30. Write a brief note on phase portrait.
31. Prove that Poisson bracket is invariant under canonical transformation.
32. Show that the generalized momentum conjugate to cyclic coordinate is conserved.
33. Define generalised Coordinates with example .Also define degrees of freedom of a system.
34. Prove that E= MC^2 in relative mechanics.
35. Let $J = \int_{x_0}^{x_1} F(y, y', x)dx$, where y is an unknown function depends on x .Derive a differential equation to find the curve $y = y(x)$ which will minimize J.

36. If a transformation from q, p to Q, P be canonical then bilinear from $\sum_i(\delta p_i dq_i - \delta q_i dp_{i)})$ remains invariant
37. Prove that the four dimensional volume element dx, dy, dz, dt is invariant under Lorentz transformation.
38. Suppose a rigid body is rotating about a fixed point. Prove that the kinetic energy is conserved throughout the motion.
39. What is Coriolis force ? Under what conditions is it equal to zero and maximum ?
40. Show that with respect to a uniformly rotating reference frame Newton's second law for a particle of mass *m* acted upon by real force **F** can be expressed as

$$\boldsymbol{F}_{eff} = \boldsymbol{F} - 2m\boldsymbol{w} \times \boldsymbol{V}rot - m\boldsymbol{w} \times (\boldsymbol{w} \times \boldsymbol{r}).$$

41. Prove that the Poisson bracket of two constants of motion is itself a constant even when the constant s depend on time explicitly
42. Prove that the phase volume is invariant under canonical transformation..
43. Prove that the linear autonomous plane systems $\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$ is stable if $p^2 - 4q > 0$, q>0 , p<0.
44. Prove that Poisson bracket is invariant under canonical transformation.
45. Use Hamilton's equation to find the equation of motion of a simple pendulum.
46. Prove that the shortest distance between two point in a plane is a straight line.
47. Derive the Lagrange's equation for conservative unconnected holonomic system.
48. Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function.
49. Show that the path followed by Particle in sliding from one point to another in the absence of friction in the shortest time is cycloid.
50. Deduce the Euler's dynamical equation when a rigid body is rotating about a fixed point.
51. Prove that $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ in relative mechanics.
52. Discuss about stability of the following system of dynamical equations $\frac{dx}{dt} = -x + y, \frac{dy}{dt} = 4x - y$
53. State the Hamilton's Principal.
54. Derived Lorentz transformation equations in special theory of relativity

55.(a) Derive the Hamilton's equations of motion from the variational principle

(b) The potential energy and kinetic energy of a dynamical system are given by

$V = \frac{kr^2}{2}$ and $T = \frac{m\dot{r}^2}{2} + \frac{mr^2\dot{\theta}^2}{2} + \frac{mr^2 \sin^2 \phi\dot{\phi}^2}{2}$. Determine the Lagrangian and Lagrange's equations of motion.

____________________________End____________________________